Set 8: Binding energy

8.1 The difference is a single neutron, so the mass difference = 1 u = 931 MeV

8.2 (a) $^{0}_{0}\gamma \rightarrow {}^{0}_{+1}e + {}^{0}_{-1}e$

(b) mass equivalent, m = electron mass + positron mass = 2 x 0.000549 u = 0.001098 u

energy equivalent, E = 0.001098 x 931MeV

= 0.001098 x 931 x 10^6eV x 1.6 x 10^{-19}J = 1.64 x 10^{-13} J

8.3 mass defect = mass of (92 protons + 143 neutrons) - mass of a uranium-235 nucleus

= (92 x 1.00728 u) + (143 x 1.00867 u) – 235.04393 u = 1.86564 u

energy equivalent, E = 1.86564 x 931 MeV = 1736.91 MeV (this is the binding energy)

so the binding energy per nucleon = 1736.91 MeV ÷ 235 = 7.39 MeV nucleon^{-1}

8.4 Binding energy per nucleon is a measure of the stability of the nucleus, so nucleus B is more stable.

8.5 (a) mass defect = mass of 3 protons + mass of 4 neutrons - mass of a lithium-7 nucleus

= (3 x 1.00728 u) + (4 x 1.00867 u) – 7.01601 u = 0.04051 u

energy equivalent, E = 0.04051 x 931 MeV = 37.7 MeV (this is the binding energy)

so the binding energy per nucleon = 37.71MeV ÷ 7 = 5.39 MeV nucleon^{-1}

(b) mass defect = mass of 53 protons + mass of 78 neutrons - mass of an iodine-131 nucleus

= (53 x 1.00728 u) + (78 x 1.00867 u) – 130.90613 u = 1.1560 u

energy equivalent, E = 1.1560 x 931 MeV = 1076.21 MeV (this is the binding energy)

so the binding energy per nucleon = 1076.21 MeV ÷ 131 = 8.22 MeV nucleon^{-1}

8.6 H-2: mass defect = mass of 1 proton + mass of 1 neutron - mass of a helium-2 nucleus

= 1.00728 u + 1.00867 u – 2.01355 u = 0.0024 u

energy equivalent, E = 0.0024 x 931 MeV = 2.23 MeV (this is the binding energy)

so the binding energy per nucleon = 2.23 MeV ÷ 2 = 1.12 MeV nucleon^{-1}

H-3: mass defect = mass of 1 proton + mass of 2 neutrons - mass of a helium-3 nucleus

= (1.00728 u) + (2 x 1.00867 u) – 3.01605u = 0.00857 u

energy equivalent, E = 0.00857 x 931 MeV = 7.98 MeV (this is the binding energy)

so the binding energy per nucleon = 7.98 MeV ÷ 3 = 2.6 MeV nucleon^{-1}

Tritium has the higher BE per nucleon

8.7 C-12: mass defect = mass of 6 protons + mass of 6 neutrons - mass of a carbon-12 nucleus

= (6 x 1.00728 u) + (6 x 1.00867 u) – 12 u = 0.0957 u

energy equivalent, E = 0.0957 x 931 MeV = 89.1 MeV (this is the binding energy)

so the binding energy per nucleon = 89.1 MeV ÷ 12 = 7.42 MeV nucleon^{-1}

C-14: mass defect = mass of 6 protons + mass of 6 neutrons - mass of a carbon-12 nucleus

= (6 x 1.00728 u) + (8 x 1.00867 u) – 14.00324 u = 0.1098 u

energy equivalent, E = 0.1098 x 931 MeV = 102.22 MeV (this is the binding energy)

so the binding energy per nucleon = 102.22 MeV ÷ 14 = 7.30 MeV nucleon^{-1}

C-12 has a greater BE per nucleon so it is more stable.

8.8 (a) a positron

(b) mass of reactants = 2 x 1.00783 u = 2.01456 u

mass of products = 2.01355 u + 0.000549 u = 2.0141 u

mass defect = 2.01456 u – 2.0141 u = 0.00046 u

= 0.00046 x 1.66054 x 10^{-27} kg = 7.64 x 10^{-31} kg

(c) E = m x c^2 = 7.64 x 10^{-31} kg x (3 x 10^8 m s^{-1})^2 = 6.875 x 10^{-14} J

= 6.875 x 10^{-14} J (÷1.60 x 10^{-13} J) = 0.430 MeV

8.9 (a) mass of reactants = 2.01355 u + 3.01605 u = 5.0285 u

mass of products = 4.00260 u + 1.00867 u = 5.0102 u

mass defect = 5.0285 u – 5.0102 u = 0.0183 u

= 0.0133 x 1.66054 x 10^{-27}kg = 3.04 x 10^{-29} kg

(b) E = m x c^2 = 3.04 x 10^{-29} kg x (3 x 10^8 m s^{-1})^2 = 2.74 x 10^{-12} J

= 2.74 x 10^{-12} J (÷1.60 x 10^{-13} J) = 17.1 MeV

8.10 mass of reactants = [238.05079 u – (92 x 0.000549u)] = 238.00028 u

mass of products = [4.00260 u – (2 x 0.000549u)] + [234.0436 u – (90 x 0.000549u)] = 237.9957 u

mass defect = 238.00028 u – 237.9957 u = 0.00458 u

= 0.00458 x 1.66054 x 10^{-27} kg = 7.6053 x 10^{-30} kg

E = m x c^2 = 7.6053 x 10^{-30} kg x (3 x 10^8 m s^{-1})^2 = 6.845 x 10^{-13} J

= 6.845 x 10^{-13} J (÷1.60 x 10^{-13} J) = 4.28 MeV

8.11 (a) - there are four neutrons released in this fission reaction

(b) mass of reactants = [(235.04393 u – (92 x 0.000549u)] + 1.00867 = 236.00209 u

mass of products = [141.92971 u – (54 x 0.000549u)]+[89.90774 u – (38 x 0.000549u)]+ (4 x 1.00867 u)

= 235.8216 u

mass defect = 236.00209 u – 235.8216 u = 0.1805 u

= 0.1805 x 1.66054 x 10^{-27}kg = 2.9973 x 10^{-28} kg

E = m x c^2 = 2.9973 x 10^{-28} kg x (3 x 10^8 m $s^{-1})^2$ = 2.698 x 10^{-11} J

= 2.698 x 10^{-11} J (÷1.60 x 10^{-13} J) = 169 MeV

(c) In a nuclear reactor the chain reaction is controlled however with a nuclear bomb, the reaction is not controlled.

8.12 (a) The nuclear reaction involves "lost" mass which is converted into energy. The products produced have huge amounts of kinetic energy, generating the thermal energy which then drives the reactor.

(b) This energy is not directed to a specific place and requires a coolant to safely take it away where it can be used to produce steam to drive turbines.

8.13 (a) $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n$

(b) mass of reactants = [(235.04393 u – (92 x 0.000549u)] + 1.00867 u = 236.00209 u

mass of products = [140.91441 u – (56 x 0.000549u)]+[91.92616 u – (36 x 0.000549u)]+ (3 x 1.00867 u)

= 235.8161 u

mass defect = 236.00209 u – 235.8161 u = 0.1860 u

= 0.1860 x 1.66054 x 10^{-27} kg = 3.0886 x 10^{-28} kg

E = m x c^2 = 3.0886 x 10^{-28} kg x (3 x 10^8 m $s^{-1})^2$ = 2.780 x 10^{-11} J

= 2.780 x 10^{-11} J (÷1.60 x 10^{-13} J) = 174 MeV

(c) mass of uranium atom = 235.04393 u x 1.66054 x 10^{-27} kg = 3.903 x 10^{-25} kg

(d) number of atoms in 1.00kg of uranium-235 = 1 kg ÷ 3.903 x 10^{-25} kg = 2.56 x 10^{24}

(e) energy per fission reaction = 174 MeV

so energy released when 1.00 kg of pure uranium-235 fissions = 2.780 x 10^{-11} J x 2.56 x 10^{24}

= 7.1 x 10^{13} J

Assuming that there is one nucleus per uranium atom and that they all undergo the fission process producing the exact same products as specified in part a).

(f) Mass of uranium required = 9.76 x 10^{13} J ÷ 7.1 x 10^{13} J = 1.37 kg

8.14 (a) $^{1}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He$

(b) mass of reactants = (2.01355 u + 1.00783 u) = 3.020282 u

mass of products = 3.01603 u

mass defect = 3.020282 u – 3.014932 u = 0.00535 u

= 0.00535 x 1.66054 x 10^{-27} kg = 8.884 x 10^{-30} kg

E = m x c^2 = 8.884 x 10^{-30} kg x (3 x 10^8 m $s^{-1})^2$ = 7.996 x 10^{-13} J

= 7.996 x 10^{-13} J (÷1.60 x 10^{-13} J) = 4.998 MeV

(c) mass of deuterium atom = 2.01355 u x 1.66054 x 10^{-27} kg = 3.34 x 10^{-27} kg

number of atoms in 1.00kg of deuterium = 1 kg ÷ 3.34 x 10^{-27} kg = 2.99 x 10^{26}

(d) 2.39 x 10^{14} J

8.15 mass of reactants = [(14.00307 u – (7 x 0.000549u)] + [(4.0026 u – (2 x 0.000549u)] = 18.000729 u

mass of products = [(16.994738 u – (8 x 0.000549u)] + 1.00728 u = 18.002018 u

mass defect = 18.000729 – 18.002018 u = -0.001289 u

= -0.001289 x 1.66054 x 10^{-27} kg = -2.14 x 10^{-30} kg

E = m x c^2 = -2.14 x 10^{-30} kg x (3 x 10^8 m $s^{-1})^2$ = -1.926 x 10^{-13} J

= -1.926 x 10^{-13} J (÷1.60 x 10^{-13} J) = -1.204 MeV

Adding on the original 3 MeV of kinetic energy possessed by the bombarding alpha particle, then the reaction products will have a total KE = 3 + (-1.204) = 1.80 MeV

8.16 (a)  - the nucleon released is a proton

(b) mass of reactants = [(14.00307 u – (7 x 0.000549u)] + 1.00867 u = 15.007897 u

mass of products = [(14.00324 u – (6 x 0.000549u)] + 1.00728 u = 15.007226 u

mass defect = 15.007897 u – 15.007226 u = 0.000671 u

= 0.000671 x 1.66054 x 10^{-27} kg = 1.11 x 10^{-30} kg

(c) E = m x c^2 = 1.11 x 10^{-30} kg x (3 x 10^8 m $s^{-1})^2$ = 9.99 x 10^{-14} J

= 9.99 x 10^{-14} J (÷1.60 x 10^{-13} J) = 0.624 MeV

(d) V = √(2 x E_k ÷ m_p) = √[(2 x 9.99 x 10^{-14} J ÷ (1.67262 x 10^{-27} kg)] = 1.09 x 10^7 m s^{-1}